CANCELLATION OF ANOMALIES IN THE SUPER-POINCARÉ COVARIANT QUANTIZATION OF THE GREEN-SCHWARZ SUPERSTRING

E.R. NISSIMOV and S.J. PACHEVA
Institute of Nuclear Research and Nuclear Energy, Boulevard Lenin 72, 1784 Sofia, Bulgaria

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We find the correct quantum modifications of the first-class hamiltonian constraints entering the super-Poincaré covariant formulation of the Green–Schwarz (GS) superstring of Nissimov, Pacheva and Solomon (harmonic GS superstring) which lead to a nilpotent quantum BRST charge. This completes the proof of the quantum consistency of the harmonic GS superstring

1. In a series of recent papers [1–4] a new physically equivalent formulation of the Green–Schwarz (GS) superstring [5,6], called the harmonic GS superstring, was developed which allows for a straightforward application of the BFV-BRST [7] procedure for a manifestly super-Poincaré covariant quantization. The basic feature of this new formulation was the introduction of:

(i) auxiliary bosonic variables ($V_{\alpha}^{\pm 1/2}$, u_{μ}^{a}) [#1] (called harmonics since they form a homogeneous space related to the "moving light-cone" homogeneous space SO(1,9)/SO(8)×SO(1,1) [8]);

(ii) additional fermionic string coordinates Ψ_A^a ($A=1, 2$) of the Ramond–Neveu–Schwarz (RNS) type;

(iii) additional gauge invariances beyond those of the original GS action.

These ingredients allowed to construct a new harmonic GS superstring action, physically equivalent to the old one but possessing BFV-irreducible (i.e. functionally independent) first-class constraints only [2,3]. Thus, the two major problems of the original GS formalism were solved:

(a) covariant disentangling of the mixture of the first-class and second-class parts of the fermionic string constraints;

(b) converting the covariantly disentangled second-class constraints into an equivalent set of first-class constraints by the Faddeev–Shatashvili method [9]. (Unless this is done, the presence of second-class constraints would break manifest space–time supersymmetry through nonzero Dirac brackets among the superstring coordinates X^{μ}, $\theta_{A\alpha}$).

In the present letter we proceed with the systematic covariant quantization of the harmonic GS superstring and prove the nilpotency of its properly quantized BRST charge $Q_{\rm BRST}$ [#2].

2. Here we briefly recall the basic formulas of the harmonic GS formalism [2–4] and point out the problems in the naive attempt to quantize it. For simplicity, in the present letter we shall restrict ourselves to the formalism where the κ-gauge symmetry (the generalization of the original fermionic κ-gauge symmetry of the GS action [5]) is covariantly gauge-fixed by $\alpha_{A\alpha}=0$ (see section 4 of ref. [3]). The reason is that this partially gauge-fixed covariant formalism captures all the difficulties in the consistent covariant quantization of the har-

[#1] The indices μ, α are $D=10$ Lorentz-vector and Majorana–Weyl spinor ones whereas the indices $a=1, ..., 8$, $\pm$ are internal SO(8)×SO(1, 1) ones, where the index a may transform under any one of the fundamental (v), (s), (c) representations of SO(8).

[#2] In particular, this result provides a refutation of the recent suspicions raised in ref. [10] about the quantum consistency of the harmonic GS formulation monic GS superstring, and the generalization to the totally unfixed formalism is straightforward.

In the covariant κ-gauge-fixed formalism, the harmonic GS action reads in hamiltonian (phase space) form [3]

$$S=S_{GS}+\hat{S}_{\text{harmonic}}\,, \tag{1}$$

$$S_{GS}=\int d\tau \int_{-\pi}^{\pi} d\xi\left(P_\mu \partial_\tau X^\mu+\sum_A (i\Psi^a_A \partial_\tau \Psi_{Aa}-\Lambda_A T_A\right)\,, \tag{2}$$

$$\hat{S}_{\text{harmonic}}=\int d\tau(p^\mu_{ua}\partial_\tau u^a_\mu+p_v^{-1/2\alpha}\,\partial_\tau v_\alpha^{+1/2}+p_v^{+1/2\alpha}\,\partial_\tau v_\alpha^{-1/2}-\Lambda_{ab}\hat{D}^{ab}-\Lambda^{+-}D^{-+}-\Lambda_a^- D^{+a}-\Lambda_a^+\hat{D}^{-a})\,. \tag{3}$$

The auxiliary variables $(v_\alpha^{\pm 1/2}, u^a_\mu)$ strongly satisfy the kinematical constraints

$$u^a_\mu u^{b\mu}=C^{ab}\,,\quad u^a_\mu(v^{\pm 1/2}\sigma^\mu v^{\pm 1/2})=0\,,\quad (v^{+1/2}\sigma^\mu v^{+1/2})(v^{-1/2}\sigma_\mu v^{-1/2})=-1\,, \tag{4}$$

where C^{ab} denotes the invariant metric tensor in the relevant SO(8) representation space. Due to the famous $D=10$ Fierz identities the composite Lorentz vectors $u^\pm_\mu\equiv v^{\pm 1/2}\sigma^\mu v^{\pm 1/2}$ are identically light-like.

In (2), (3) $\Lambda_A(\xi), \Lambda_{ab}, ..., \Lambda_a^+$ are Lagrange multipliers for the corresponding first-class constraints:

$$T_A\equiv \mathscr{P}_A^2+2i(-1)^A\Psi^a_A\Psi'_{Aa}\,, \tag{5}$$

$$\hat{D}^{ab}\equiv D^{ab}+\sum_A\int_{-\pi}^{\pi} d\xi\,\tilde{R}^{ab}_A\,, \tag{6}$$

$$D^{-+}\equiv\frac{1}{2}\left(v_\alpha^{+1/2}\frac{\partial}{\partial v_\alpha^{+1/2}}-v_\alpha^{-1/2}\frac{\partial}{\partial v_\alpha^{-1/2}}\right),\quad D^{+a}\equiv u^+_\mu\frac{\partial}{\partial u_{\mu a}}+\tfrac{1}{2}v^{-1/2}\sigma^+\sigma^a\frac{\partial}{\partial v^{-1/2}}\,, \tag{7,8}$$

$$\hat{D}^{-a}\equiv D^{-a}-\sum_A\int_{-\pi}^{\pi} d\xi(\mathscr{P}_A^+)^{-1}\mathscr{P}_{Ac}\tilde{R}^{ac}_A\,. \tag{9}$$

In eqs. (5)–(9) the following notations are used:

$$D^{ab}\equiv u^a_\mu\frac{\partial}{\partial u_{\mu b}}-u^b_\mu\frac{\partial}{\partial u_{\mu a}}+\frac{1}{2}\left(v^{+1/2}\sigma^{ab}\frac{\partial}{\partial v^{+1/2}}+v^{-1/2}\sigma^{ab}\frac{\partial}{\partial v^{-1/2}}\right),\quad D^{-a}\equiv u^-_\mu\frac{\partial}{\partial u_{\mu a}}+\tfrac{1}{2}v^{+1/2}\sigma^-\sigma^a\frac{\partial}{\partial v^{+1/2}}\,,$$

$$\mathscr{P}^\mu_A(\xi)\equiv P^\mu(\xi)+(-1)^A X^{\mu\prime}(\xi)=\frac{1}{\sqrt{\pi}}\sum_{n=-\infty}^{\infty}{}^{(}\tilde{\alpha}^{)}{}^\mu_n\exp(\pm in\xi)\,, \tag{10}$$

$$\tilde{R}^{ab}_A(\xi)\equiv\tfrac{1}{2}(\tilde{S}^{ab})_{cd}\Psi^c_A(\xi)\Psi^d_A(\xi)\,, \tag{11}$$

$$(\tilde{S}^{ab})_{cd}\equiv\tfrac{1}{2}v^{-1/2}\sigma_c\sigma^{ab}\sigma^+\sigma_d v^{-1/2}\,,\quad \sigma^{ab}\equiv\sigma^{[a}\sigma^{b]}\,,\quad \sigma^a\equiv u^a_\mu\sigma^\mu\,,\quad \sigma^\pm\equiv u^\pm_\mu\sigma^\mu\,,\quad \mathscr{P}_A^\pm\equiv\mathscr{P}^\mu_A u^\pm_\mu\,. \tag{12}$$

The 8×8 matrices $\tilde{S}^{ab}$ (12) are precisely the $D=10$ Lorentz-invariant generators of the harmonic SO(8) (c)-spinor representation [3,4].

The classical Poisson-bracket (PB) algebra of the constraints (5)–(9) reads

$$\{T_A(\xi), T_B(\eta)\}_{PB}=8(-1)^A\delta_{AB}[T_A(\xi)\delta'(\xi-\eta)+\tfrac{1}{2}T'_A(\xi)\delta(\xi-\eta)]\,,\quad \{T_A(\xi),\hat{D}^{-a}\}_{PB}=0\,, \tag{13,14}$$

$$-i\{\hat{D}^{-a},\hat{D}^{-b}\}_{PB}=\sum_A\int_{-\pi}^{\pi} d\xi(\mathscr{P}_A^+)^{-2}\tilde{R}^{ab}_A T_A\,,\quad \{T_A(\xi),(\hat{D}^{ab}, D^{-+}, D^{+a})\}_{PB}=0\,, \tag{15,16}$$

$$\mathrm{i}\{\hat{\mathrm{D}}^{ab},\hat{\mathrm{D}}^{-c}\}_{\mathrm{PB}}=C^{bc}\hat{\mathrm{D}}^{-a}-C^{ac}\hat{\mathrm{D}}^{-b}\,,\quad \mathrm{i}\{\hat{\mathrm{D}}^{ab},\hat{\mathrm{D}}^{cd}\}_{\mathrm{PB}}=C^{bc}\hat{\mathrm{D}}^{ad}-C^{ac}\hat{\mathrm{D}}^{bd}+C^{ad}\hat{\mathrm{D}}^{bc}-C^{bd}\hat{\mathrm{D}}^{ac}\,, \tag{17,18}$$

$$\mathrm{i}\{\hat{\mathrm{D}}^{ab},\mathrm{D}^{+c}\}_{\mathrm{PB}}=C^{bc}\mathrm{D}^{+a}-C^{ac}\mathrm{D}^{+b}\,,\quad \{\hat{\mathrm{D}}^{ab},\mathrm{D}^{-+}\}_{\mathrm{PB}}=0\,, \tag{19}$$

$$\mathrm{i}\{\mathrm{D}^{-+},\mathrm{D}^{+a}\}_{\mathrm{PB}}=\mathrm{D}^{+a}\,,\quad \mathrm{i}\{\mathrm{D}^{-+},\hat{\mathrm{D}}^{-a}\}_{\mathrm{PB}}=-\hat{\mathrm{D}}^{-a}\,,\quad \mathrm{i}\{\mathrm{D}^{+a},\hat{\mathrm{D}}^{-b}\}_{\mathrm{PB}}=C^{ab}\mathrm{D}^{-+}+\hat{\mathrm{D}}^{ab}\,. \tag{20,21,22}$$

We now proceed to the quantization of the constraints (5)–(9) and their algebra (13)–(22). Clearly, difficulties may arise only in the quantization of $T_A(\xi)$ (5) and $\hat{\mathrm{D}}^{-a}$ (9) and their respective algebra (13)–(16).

Let us first consider $T_A(\xi)$ (5) and quantize it by simple normal ordering. Then the quantized version of (13) reads

$$[:T_A(\xi):,:T_B(\eta):]=8\mathrm{i}(-1)^A\delta_{AB}[:T_A(\xi):\,\delta'(\xi-\eta)+\tfrac{1}{2}\,:T'_A(\xi):\,\delta(\xi-\eta)-\alpha_0\delta'''(\xi-\eta)]\,, \tag{23}$$

$$\alpha_0\equiv(1/12\pi)(10\times1+8\times\tfrac{1}{2})=14/12\pi\,. \tag{24}$$

Thus, the naive quantization of $T_A(\xi)$ (5) would lead to conformal anomalies since the α_0 term in (23) cannot be cancelled by the conformal ghost contribution: $\alpha_0(\text{ghost})=-26/12\pi$.

Next, calculating the quantum commutator corresponding to (15) with straightforwardly quantized (just normal-ordered) $\hat{\mathrm{D}}^{-a}$ (9) we obtain an operator anomaly:

$$[\hat{\mathrm{D}}^{-a},\hat{\mathrm{D}}^{-b}]=-\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\,:(\mathcal{P}_A^+)^{-2}\tilde{R}_A^{ab}T_A:-\frac{\mathrm{i}}{4\pi}\sum_A(-1)^A\int_{-\pi}^{\pi}\mathrm{d}\xi(\mathcal{P}_A^+)^{-2}[:\mathcal{P}_A^a(\mathcal{P}_A^b)'-(\mathcal{P}_A^a)'\mathcal{P}_A^b:]$$
$$-\frac{3}{\pi}\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\,\Delta_A^{--ab}(\xi)\,, \tag{25}$$

$$\Delta_A^{--ab}(\xi)\equiv(\mathcal{P}_A^+)^{-2}\left\{\tfrac{1}{2}(\tilde{S}^{ab})_{cd}\Psi_A'^c\Psi_A'^d+\tilde{R}_A^{ab}\left[\left(\frac{\mathcal{P}_A^{+\prime}}{\mathcal{P}_A^+}\right)^2-\tfrac{1}{2}\mathcal{P}_A^{+2}\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\left(\frac{1}{\mathcal{P}_A^{+2}}\right)\right]\right\}\,. \tag{26}$$

The first operator anomaly term on the RHS of (25) comes from the Kač–Moody anomaly of the SO(8) generators (11):

$$[\tilde{R}_A^{ab}(\xi)\,,\tilde{R}_B^{cd}(\eta)]=\delta_{AB}[C^{bc}\tilde{R}_A(\xi)-C^{ac}\tilde{R}_A^{bd}(\xi)+C^{ad}\tilde{R}_A^{bc}(\xi)-C^{bd}\tilde{R}_A^{ac}(\xi)]\delta(\xi-\eta)$$
$$-(-1)^A\delta_{AB}(\mathrm{i}/2\pi)(C^{ac}C^{bd}-C^{ad}C^{bc})\delta'(\xi-\eta)\,. \tag{27}$$

The second anomaly term (26) on the RHS of (25) arises from Wick contractions in the normal-ordering process:

$$\Delta_A^{--ab}(\xi)=\int_{-\pi}^{\pi}\mathrm{d}\eta\left(\frac{\overbrace{\mathcal{P}_{Ac}(\xi)\mathcal{P}_{Ad}(\eta)}}{\mathcal{P}_A^+(\xi)\mathcal{P}_A^+(\eta)}[\tilde{R}_A^{ac}(\xi),\tilde{R}_A^{bd}(\eta)]+2\mathrm{i}(-1)^A\frac{\delta'(\xi-\eta)}{\mathcal{P}_A^+(\xi)\mathcal{P}_A^+(\eta)}\overbrace{\tilde{R}_A^{ac}(\xi)\tilde{R}_{Ac}^b(\eta)}\right)$$
$$=-3\mathrm{i}(-1)^A\int_{-\pi}^{\pi}\mathrm{d}\eta\frac{(\tilde{S}^{ab})_{cd}\Psi_A^c(\xi)\Psi_A^d(\eta)}{\mathcal{P}_A^+(\xi)\mathcal{P}_A^+(\eta)}\frac{\mathrm{d}}{\mathrm{d}\xi}\{\delta(\xi-\eta)[\omega(\pm(\xi-\eta))-\omega(\mp(\xi-\eta))]\}\,, \tag{28}$$

where $\omega(\pm(\xi-\eta))\equiv\omega((-1)^{A+1}(\xi-\eta))$ is the Wick contraction function:

$$\overbrace{\Psi_A^a(\xi)\Psi_A^b}(\eta)=C^{ab}\omega(\pm(\xi-\eta))\,,\quad \overbrace{\mathcal{P}_A^\mu(\xi)\mathcal{P}_A^\nu}(\eta)=2\mathrm{i}(-1)^A\eta^{\mu\nu}\frac{\mathrm{d}}{\mathrm{d}\xi}\omega(\pm(\xi-\eta))\,,$$

$$\omega(\xi-\eta)\equiv\frac{1}{2\pi}\left(\sum_{n=1}^{\infty}\exp[\mathrm{i}n(\xi-\eta)]+\tfrac{1}{2}\right)\,. \tag{29}$$

Regularization of the product of singular functions in the last line of eq. (28) uses the following simple identities (cf. definition (29)):

$$\delta(\xi-\eta)=\omega(\xi-\eta)+\omega(\eta-\xi)\,,\quad \omega^2(\xi-\eta)-\omega^2(\eta-\xi)=-(\mathrm{i}/2\pi)\delta'(\xi-\eta)\,. \tag{30}$$

Inserting (30) into (28) yields the result (26).

We end up this section with the conclusion that the naive quantization of $T_A(\xi)$ (5) and $\hat{\mathrm{D}}^{-a}$ (9) leads both to conformal and harmonic anomalies. Let us particularly stress that exactly the same situation occurs in the Lorentz non-covariant canonical quantization procedure for the GS and RNS superstrings when only the fermionic gauge symmetries are fixed [#3] but the ordinary reparametrization invariance is left intact. In this latter case the analog of $\hat{\mathrm{D}}^{-a}$ is the Lorentz-boost generator J^{-a} (cf. section 4 of ref. [3]).

3. The solution to the anomaly problem proceeds along the following steps:

(i) The modification $\tilde{T}_A(\xi)$ of $:T_A(\xi):$ (improved energy–momentum tensor) by adding an appropriate local operator (a total derivative in order to preserve the string spectrum) such that the Virasoro algebra (23) is preserved modulo a shift of the anomaly coefficient α_0 (24) to the value $26/12\pi$ needed for cancellation of the conformal anomalies in Q_{BRST}.

(ii) The modification $\mathscr{D}^{-a}$ of the quantized harmonic generator $\hat{\mathrm{D}}^{-a}$ by adding an appropriate integrated local operator such that the quantum version of (14)

$$[\tilde{T}_A(\xi)\,,\mathscr{D}^{-a}]=0 \tag{31}$$

holds.

(iii) The modifications $\tilde{T}_A(\xi)$ and $\mathscr{D}^{-a}$ should be such that the quantum commutation relations corresponding to (17)–(22) remain the same.

(iv) The modification $\mathscr{D}^{-a}$ should be such that the possible anomaly in $[\mathscr{D}^{-a}, \mathscr{D}^{-b}]$ is cancelled by the compensating anomalous contribution of the ghosts in Q_{BRST}.

We find the following solutions to steps (i)–(iii) above:

$$\tilde{T}_A(\xi)=\,:T_A(\xi):\,+f_A\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\mathscr{P}_A^{+\prime}}{\mathscr{P}_A^{+}}\right),\quad \mathscr{D}^{-a}=\hat{\mathrm{D}}^{-a}+\frac{1}{4}\sum_A(-1)^A f_A\int_{-\pi}^{\pi}\mathrm{d}\xi\,\frac{\mathscr{P}_A^{a\prime}}{\mathscr{P}_A^{+}}(\xi)\,, \tag{32,33}$$

where f_A is a numerical constant. Indeed, we have

$$[\tilde{T}_A(\xi),\tilde{T}_B(\eta)]=8\mathrm{i}(-1)^A\delta_{AB}[\tilde{T}_A(\xi)\delta'(\xi-\eta)+\tfrac{1}{2}\tilde{T}'_A(\xi)\delta(\xi-\eta)-\alpha(f)\delta'''(\xi-\eta)]\,, \tag{34}$$

$$\alpha(f)\equiv\alpha_0-f_A=14/12\pi+1/\pi=26/12\pi\,, \tag{35}$$

for the particular choice $f_A=-1/\pi$. Further, eq. (31) is straightforwardly verified inserting the expressions (32), (33).

Let us particularly note that (32), (33) represent the unique solution to tasks (i)–(iii). In fact, there is one more admissible addition to $:T_A(\xi):$ fulfilling (i) – the operator $g_A\,\mathrm{d}(\mathscr{P}_A^{+\prime\prime}/\mathscr{P}_A^{+\prime})/\mathrm{d}\xi$ with g_A being a numerical constant. It, however, either breaks (31) (if $g_A\neq-2f_A$) or it does not shift the central charge α_0 in (23) (if $g_A=-2f_A$) and, therefore, cannot cancel the conformal anomalies.

The final step is to compute the modified quantum commutator corresponding to (25). We get

[#3] (i) Fixing the κ-invariance by $\sigma^+\theta_A=0$ for GS, (ii) fixing the fermionic part of the superconformal invariance by $\Psi_A^+=0$ for RNS. Here in both cases + denotes the non-covariant $D=10$ light-cone index

$$[\mathscr{D}^{-a},\mathscr{D}^{-b}]=-\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\,{:}(\mathscr{P}_A^+)^{-2}\tilde{R}_A^{ab}\tilde{T}_A{:}+\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\left\{f_A\frac{\tilde{R}_A^{ab}}{\mathscr{P}^{+2}}\left[2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\mathscr{P}^{+\prime}}{\mathscr{P}_A^+}\right)-\left(\frac{\mathscr{P}_A^{+\prime}}{\mathscr{P}_A^+}\right)^2\right]-\frac{3}{\pi}\Delta_A^{--ab}(\xi)\right\}$$

$$-\frac{\mathrm{i}}{4}\sum_A(-1)^A(1/\pi+f_A)\int_{-\pi}^{\pi}\mathrm{d}\xi\frac{1}{\mathscr{P}_A^{+2}}[{:}\mathscr{P}_A^a(\mathscr{P}_A^b)'-(\mathscr{P}_A^a)'\mathscr{P}_A^b{:}]$$

$$=-\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\,{:}\frac{\tilde{R}_A^{ab}}{\mathscr{P}_A^{+2}}\tilde{T}_A{:}-\frac{1}{\pi}\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\left\{\frac{\tilde{R}_A^{ab}}{\mathscr{P}_A^{+2}}\left[2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\mathscr{P}_A^{+\prime}}{\mathscr{P}_A^+}\right)-\left(\frac{\mathscr{P}_A^{+\prime}}{\mathscr{P}_A^+}\right)^2\right]+3\Delta_A^{--ab}(\xi)\right\}. \tag{36}$$

Comparing (34), (35) and (36), we see the following remarkable feature. The choice $f_A=-1/\pi$ in (32) cancels simultaneously both the conformal anomalies and the Kač–Moody anomalies in $[\mathscr{D}^{-a},\mathscr{D}^{-b}]$ (36). Let us particularly stress here the role of the space–time dimension $D=10$. If we consider a different $D\neq 10$ (where the GS superstring exists on the classical level) we can always adjust the coefficient f_A in (32) such that the conformal anomalies cancel also in $D\neq 10$. However, the coefficient of the Kač–Moody anomaly in (27) is universal for all $\mathrm{SO}(D-2)$, hence, cancelling of the conformal anomalies in $D\neq 10$ would retain the harmonic Kač–Moody anomalies or vice versa.

What about the operator anomaly still present in the last line of eq. (36)? It turns out that this anomaly is cancelled by the compensating anomalous contribution of the ghosts in Q_{BRST}:

$$Q_{\mathrm{BRST}}=Q_{\mathrm{string}}+Q_{\mathrm{harmonic}}, \tag{37}$$

$$Q_{\mathrm{string}}=\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\,C_A(\xi)\left(\tilde{T}_A(\xi)-4\mathrm{i}(-1)^AC_A'(\xi)\frac{\delta}{\delta C_A}(\xi)\right), \tag{38}$$

$$Q_{\mathrm{harmonic}}=\mathrm{i}\eta_{ab}\left(\hat{\mathrm{D}}^{ab}+\eta^{+a}\frac{\partial}{\partial\eta_b^+}-\eta^{+b}\frac{\partial}{\partial\eta_a^+}+\eta^{-a}\frac{\partial}{\partial\eta_b^-}-\eta^{-b}\frac{\partial}{\partial\eta_a^-}+\eta_d^a\frac{\partial}{\partial\eta_{bd}}-\eta_d^b\frac{\partial}{\partial\eta_{ad}}\right)$$

$$+\mathrm{i}\eta^{+-}\left(\mathrm{D}^{-+}+\eta_a^+\frac{\partial}{\partial\eta_a^+}-\eta_a^-\frac{\partial}{\partial\eta_a^-}\right)+\mathrm{i}\eta_a^-\mathrm{D}^{+a}$$

$$+\mathrm{i}\eta_a^+\left(\hat{\mathrm{D}}^{-a}+\eta^{-a}\frac{\partial}{\partial\eta^{+-}}-\eta_b^-\frac{\partial}{\partial\eta_{ab}}+\tfrac{1}{2}\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi(\mathscr{P}_A^+)^{-2}\eta_b^+\tilde{R}_A^{ab}\frac{\delta}{\delta C_A(\xi)}\right). \tag{39}$$

Indeed, squaring Q_{BRST} (37)–(39) we get

$$Q_{\mathrm{BRST}}^2=\tfrac{1}{2}\eta_a^+\eta_b^+\left([\mathscr{D}^{-a},\mathscr{D}^{-b}]+\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\,\tfrac{1}{2}\{(\mathscr{P}_A^+)^{-2}\tilde{R}_A^{ab},\tilde{T}_A\}\right)$$

$$=-\frac{1}{2\pi}\eta_a^+\eta_b^+\sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\left\{\frac{\tilde{R}_A^{ab}}{\mathscr{P}_A^{+2}}\left[2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\mathscr{P}_A^{+\prime}}{\mathscr{P}_A^+}\right)-\left(\frac{\mathscr{P}_A^{+\prime}}{\mathscr{P}_A^+}\right)^2\right]+3[\Delta_A^{--ab}(\xi)+\tilde{\Delta}_A^{--ab}(\xi)]\right\}, \tag{40}$$

$$\tilde{\Delta}_A^{--ab}(\xi)\equiv\frac{1}{2\mathscr{P}_A^{+2}}(\tilde{S}^{ab})_{cd}\Psi_A^c(\Psi_A^d)''-\frac{2}{3}\frac{\tilde{R}_A^{ab}}{\mathscr{P}_A^{+3}}(\mathscr{P}_A^+)''. \tag{41}$$

The anomalous term (41) arises as a result of normal-ordering of the ghost contribution to Q_{BRST}^2:

$$\frac{1}{2}\left\{\frac{\tilde{R}^{ab}_A}{\mathscr{P}^{+2}_A}(\xi),\tilde{T}_A(\xi)\right\}$$

$$=\int_{-\pi}^{\pi}\mathrm{d}\eta\,\delta(\xi-\eta)\left(:\frac{\tilde{R}^{ab}_A}{\mathscr{P}^{+2}_A}(\xi)\tilde{T}_A(\eta):-\mathrm{i}(-1)^A\frac{(\tilde{S}^{ab})_{cd}}{\mathscr{P}^{+2}_A(\xi)}\Psi^c_A(\xi)\,\Psi^d_A(\eta)\frac{\overleftrightarrow{\mathrm{d}}}{\mathrm{d}\eta}[\omega(\pm(\xi-\eta))-\omega(\mp(\xi-\eta))]\right.$$

$$\left.-4\mathrm{i}(-1)^A\frac{\tilde{R}^{ab}_A}{\mathscr{P}^{+3}_A}(\xi)\mathscr{P}^+_A(\eta)\frac{\mathrm{d}}{\mathrm{d}\xi}[\omega(\pm(\xi-\eta))-\omega(\mp(\xi-\eta))]\right). \tag{42}$$

Regularization of (42) is performed by accounting for the short-distance behaviour of (29):

$$\omega(\xi-\eta)\underset{\eta\to\xi}{\approx}\frac{\mathrm{i}}{2\pi}(\xi-\eta)^{-1},$$

and expanding the operators depending on η in powers of $(\xi-\eta)$ around the point ξ to arrive at the result (41).

Now, it is straightforward to check that upon substituting in (40) the explicit expressions (26) and (41) we obtain a complete cancellation of all anomalous terms (coming from the operator anomaly in (36) and from the ghosts):

$$Q^2_{\mathrm{BRST}}=0. \tag{43}$$

This completes the proof of the quantum consistency of the harmonic GS superstring [1–4].

4. As a final comment, let us point out that the recently proposed covariant approach to the quantized GS superstring by Kallosh and Rahmanov (KR) [11,10], which employs essentially the same auxiliary variables (u, v) introduced by the harmonic GS formalism [1–3], looks more promising for calculations within the Polyakov functional integral formalism. On the other hand, it does not seem appropriate for covariant operator (canonical) quantization of the GS superstring because of the following serious problem of KR arising already in the zero-mode (superparticle) limit.

Among the harmonic constraints of KR, denoted in refs. [11,10] by $\{H, F, K\}$, there is only one, $H^{-+}\equiv D^{-+}$ (see eq. (7) above), which acts nontrivially on the harmonic light-like vector $u^+_\mu\equiv v^{+1/2}\sigma_\mu v^{+1/2}$. Then it is an elementary exercise to show that the general solution to the Dirac constraint equations for the superfield wave function $\phi=\phi(x,\theta,u,v)$ in the KR formalism

$$\{H\}\phi=0,\quad \{F\}\phi=0,\quad \{K\}\phi=0, \tag{44}$$

has the following form in momentum space representation:

$$\tilde{\phi}(p,\theta,u,v)=\sum_{\{\lambda\}}\left(\frac{u^+_{\lambda_1}}{p^+}\right)...\left(\frac{u^+_{\lambda_n}}{p^+}\right)\phi^{\{\lambda\}}(p,\theta), \tag{45}$$

where $p^+\equiv u^+_\mu p^\mu\equiv v^{+1/2}\not{p}v^{+1/2}$ and $\phi^{\{\lambda\}}(p,\theta)$ are an infinite number of arbitrary ordinary superfields which apparently do not contain among themselves the $D=10$ super-Yang–Mills connections $A^\mu(x,\theta)$ and $A^\alpha(x,\theta)$ subject to the Nilsson constraints.

Eq. (45) tells us that the KR harmonic superparticle actually describes an infinite number of unphysical supermultiplets, unlike the harmonic $N=1$ superparticle in our formalism which correctly yields on-shell the (linearized) super-Yang–Mills multiplet in $D=10$ [4] which is precisely the physical result obtained in the non-covariant light-cone gauge (see ref. [6], Vol. 2, pp. 238, 239).

Eq. (45) is the statement about the breaking (on the first-quantized level) of the pure-gauge property of the auxiliary harmonic variables in the KR approach [11,10].

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